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**MATHEMATICS**

**9709/33**

Paper 3 Pure Mathematics 3 (P3)

**October/November 2017**

**1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



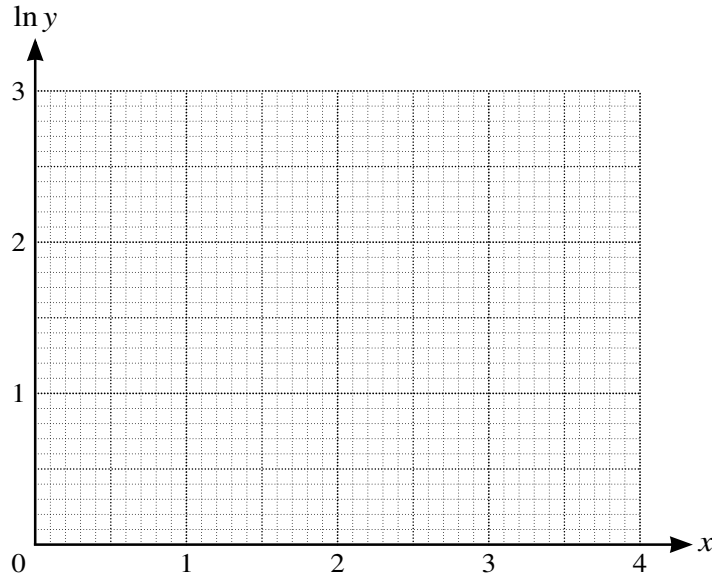
- 1** Find the quotient and remainder when  $x^4$  is divided by  $x^2 + 2x - 1$ . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 2 Two variable quantities  $x$  and  $y$  are believed to satisfy an equation of the form  $y = C(a^x)$ , where  $C$  and  $a$  are constants. An experiment produced four pairs of values of  $x$  and  $y$ . The table below gives the corresponding values of  $x$  and  $\ln y$ .

|         |     |     |     |     |
|---------|-----|-----|-----|-----|
| $x$     | 0.9 | 1.6 | 2.4 | 3.2 |
| $\ln y$ | 1.7 | 1.9 | 2.3 | 2.6 |

By plotting  $\ln y$  against  $x$  for these four pairs of values and drawing a suitable straight line, estimate the values of  $C$  and  $a$ . Give your answers correct to 2 significant figures. [5]



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**3** The equation  $x^3 = 3x + 7$  has one real root, denoted by  $\alpha$ .

**(i)** Show by calculation that  $\alpha$  lies between 2 and 3.

[2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



**4** (i) Prove the identity  $\tan(45^\circ + x) + \tan(45^\circ - x) \equiv 2 \sec 2x$ . [4]

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (ii) Hence sketch the graph of  $y = \tan(45^\circ + x) + \tan(45^\circ - x)$  for  $0^\circ \leq x \leq 90^\circ$ . [3]

**5** The equation of a curve is  $2x^4 + xy^3 + y^4 = 10$ .

(i) Show that  $\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$ . [4]

[illegible]

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**6** The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = 4 \cos^2 y \tan x,$$

for  $0 \leq x < \frac{1}{2}\pi$ , and  $x = 0$  when  $y = \frac{1}{4}\pi$ . Solve this differential equation and find the value of  $x$  when  $y = \frac{1}{3}\pi$ . [8]

[illegible]



- 7 (a) The complex number  $u$  is given by  $u = 8 - 15i$ . Showing all necessary working, find the two square roots of  $u$ . Give answers in the form  $a + ib$ , where the numbers  $a$  and  $b$  are real and exact. [5]

[illegible]

- (b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities  $|z - 2 - i| \leq 2$  and  $0 \leq \arg(z - i) \leq \frac{1}{4}\pi$ . [4]

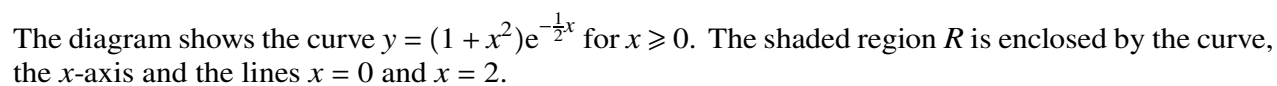
**8** Let  $f(x) = \frac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}$ .

(i) Express  $f(x)$  in the form  $A + \frac{B}{x+2} + \frac{C}{2x-1}$ . [4]

[illegible]

**(ii)** Hence show that  $\int_1^4 f(x) \, dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$ . [5]

[illegible]



- [illegible]

(ii) Show that the exact value of the area of  $R$  is  $18 - \frac{42}{e}$ .

[5]

[illegible]

- 10** The equations of two lines  $l$  and  $m$  are  $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$  and  $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$  respectively.

(i) Show that the lines do not intersect. [3]

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(ii) Calculate the acute angle between the directions of the lines. [3]

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[illegible]

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